Upper limit on lot percent defective given a sample with no defectives.

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Abstract

When a sample of size *n* is randomly selected from a lot of size *N* (N Large), there is a finite probability that the sample will contain no defective pieces. This paper addresses the question of what inferences may be made about the lot percent defective in this instance.

Since the probability of finding no defective units is the complementary event to finding at least one defective, determining the sample size containing at least one defective, given the lot proportion defective involves a simple rewrite of the equation.

Introduction / Discussion:

There is a certain niche of statistical distributions referred to as "waiting time" distributions which answer questions of the sort: "When (at what sample) will the first defect occur?"; "How long until the next person enters the queue?", etc.. Exponential, Poisson, Pascal, geometric, and binomial distributions can fall into this category when addressing particular situations.

In this case, the Poisson distribution will be used to determine the upper limit on the percent defective $100p$ for various confidence levels γ when there are no defectives in a sample of size n, subject to the following assumptions:

- removing samples does not significantly alter the underlying distribution,
- the hypergeometric distribution[1] or the binomial distribution[2] may be approximated by the Poisson distribution when the proportion defective is less than .10, and the sample size (n) is large (typically > 30 – 50). This is easily verified by passing simultaneous limits as $p \to 0$ and $n \to \infty$.

The desired relationship is:

$$\gamma = 1 - e^{-np} \qquad (1)$$

Where, n is the sample size, p is the upper limit for the proportion defective, and γ is the confidence level. Rewriting (1) in terms of the upper limit for the proportion defective yields:

$$p = \ln(1-\gamma)/(-n) \qquad (2)$$

Given p in equation (2), the equation necessary to compute the sample size containing at least one defective becomes:

$$n = \ln(1-\gamma)/(-p) \qquad (3)$$

Figures 1 and 2 provide graphic displays of equations 1 and 2, respectively, with the lines corresponding to γ = 0.90, 0.95, and 0.99.

1 Sampling is conducted without replacement.
2 Sampling is conducted with replacement.

An interesting corollary is that an upper limit for the percentage defective for a continuous process can be approximated and updated while units are undergoing testing. For example, estimate the process (simply defined as a lot of size $N \rightarrow \infty$) percent defective of 15 units that have been on test for 20 hours with no failures.

1. Substitute (20*15) = 300 for n in equation (2).
2. Compute the 95% upper limit for the failure rate as 0.01 failure/time-unit.

References:
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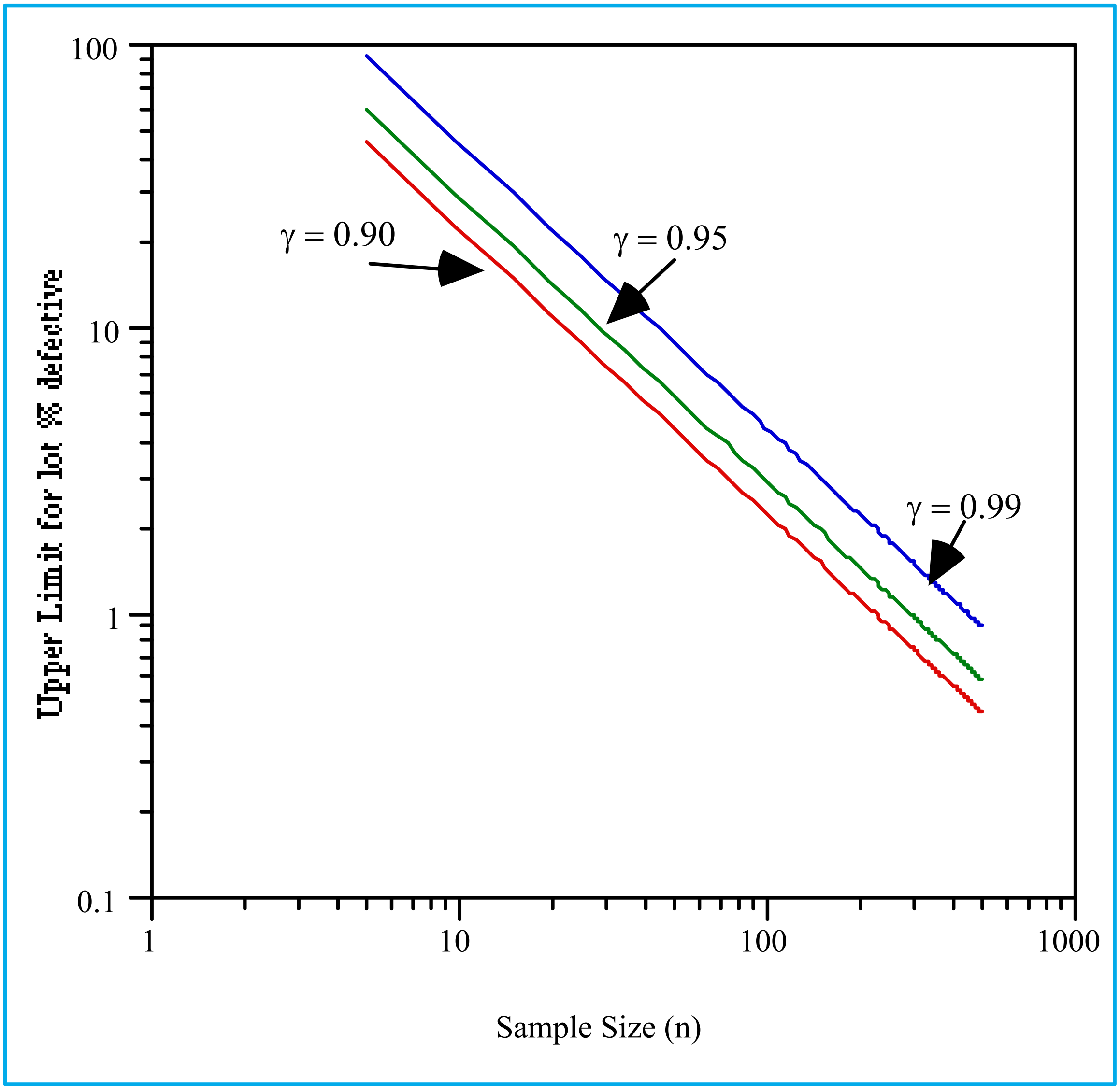

100
10
1
0.1
1
10
100
1000
Upper Limit for lot % defective
Sample Size (n)
γ = 0.90
γ = 0.95
γ = 0.99

Figure 1

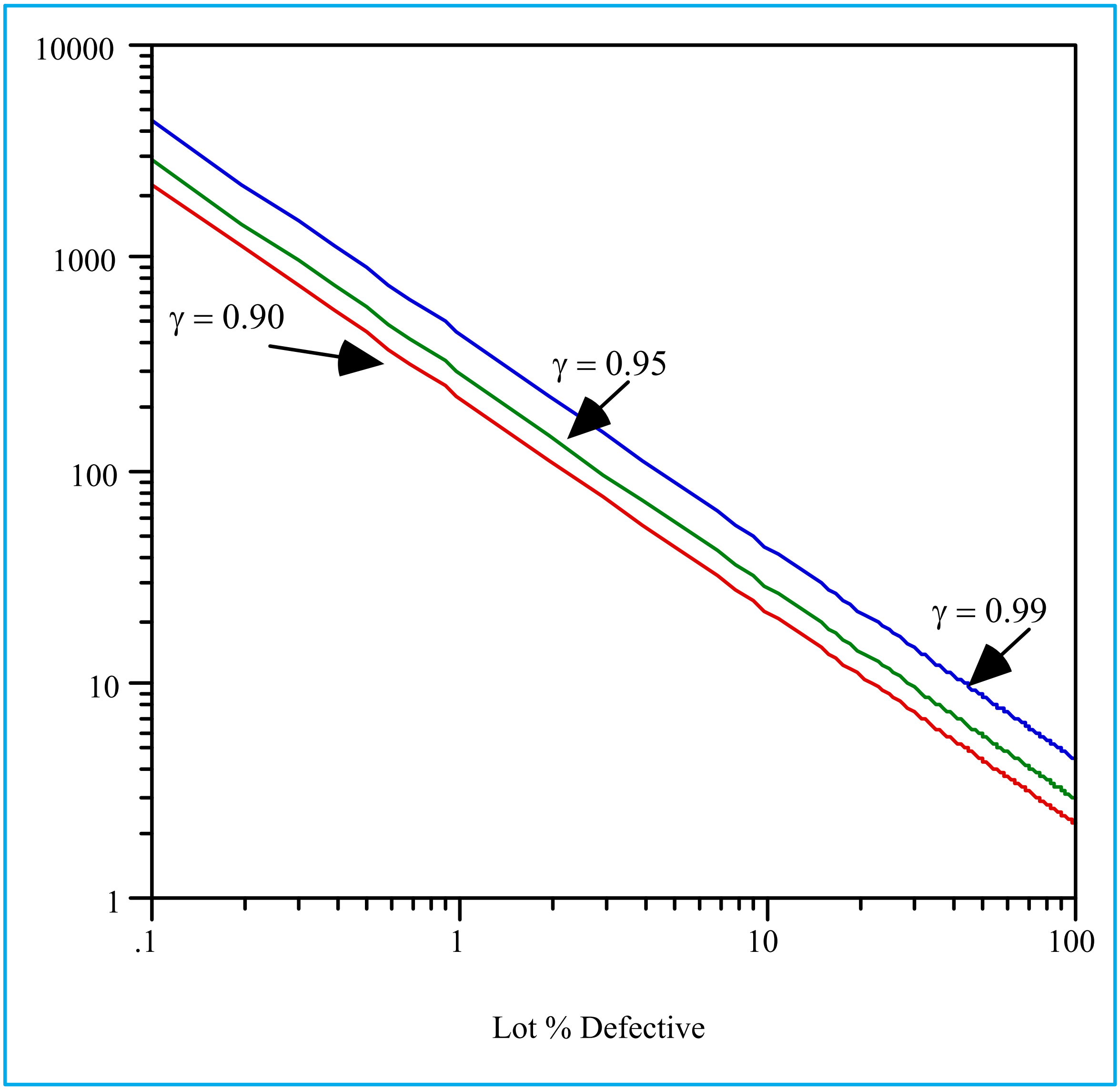

Figure 2